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Obtaining a series of Exact Solutions to Charge Density Wave Equations in Conductive Polymer

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Abstract

The weak pinning charge density wave equations of originated in the conductive polymer are investigated. Based on the trigonometric function transform method, with the aid of symbolic computation in Matlab software, we successfully obtain a series of exact solutions (including four kinds of soliton solutions and plural form exact solutions) for weak pinning charge density wave equations of originated in the conductive polymer. In addition, some new trigonometric function solutions are found

Keywords: The conductive polymer; the weak pinning charge density wave equations; symbolic computation; the trigonometric function transform method; Wu elimination method; exact solutions..

1. Introduction

. Recently, the study of nonlinear problems is of crucial importance in all areas of nonlinear engineering and materials sciences. The nonlinear complex phenomena are related to nonlinear evolution differential equation or nonlinear coupled evolution equations. Such as, weak pinning charge density wave equations of originated in the conductive polymer [1], namely

$$\rho_{tt} - c^2\rho_{xx} = (a-4e)\rho - b\rho^3 - b\rho\sigma^2 ; \sigma_{tt} - c^2\sigma_{xx} = a\sigma - b\sigma^3 - b\sigma\rho^2 , \qquad (1)$$

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Where a, b, c, e, are parameters. The nonlinear coupled evolution equations are relevant to the weak pinning charge density wave equation of originated in the conductive polymer, which were first study by S, Sarker et al [1]. Liu obtained some soliton solutions of equations (1) by double parameter hypothesis [2] . However, the construction of the exact solutions of nonlinear evolution differential equation or nonlinear coupled evolution equations is one of the most important and essential tasks in nonlinear science and engineering. In recent decades, many various powerful methods to construct exact solutions have been proposed, such as the homogeneous balance method[3], tanh method [4–7], sine–cosine method [8, 9]. In this paper, by using the trigonometric function transform method [10, 11], we seek the exact traveling wave solutions for weak pinning charge density wave equations(1) of originated in the conductive polymer. The rest of this paper is organized as follows. In Section 2, we simply introduce the trigonometric function transform method and procedures. In Section 3, applying the method, with the aid of symbolic computation in Matlab software, a series of exact solutions are obtained for charge density wave equations in conductive polymer. The last section is devoted to a short summary and conclusion.

2. Methods and procedures

In Ref. [10, 11], we present a trigonometric function transform method to construct exact solutions of nonlinear coupled differential equations. The main idea in this method is that the solutions we are looking for are the trigonometric function polynomial, which simply procedures as follows. For a given nonlinear differential evolution equation, say, in two variables,

$$F(\sigma,\rho,\sigma_t,\rho_t,\sigma_x,\rho_x,\sigma_{xx},\rho_{xx},\cdots)=0 \tag{2}$$

Step 1: We first make the transformation

$$\sigma(x,t)=\sigma(\xi),\rho(x,t)=\rho(\xi),\xi=x-ut \tag{3}$$

then Eq.(2) becomes a nonlinear ordinary differential equation

$$F_0(\sigma,\rho,\sigma_\xi,\rho_\xi,\sigma_{\xi\xi},\rho_{\xi\xi},\cdots)=0 \tag{4}$$

Step 2: The next crucial step is we assume Eq.(4) has the following solutions

$$\sigma(\xi)=A_0+\sum_{l=1}^{m}(A_l\cos^l k\varphi+B_l\sin^l k\varphi);\rho(\xi)=C_0+\sum_{j=1}^{n}(C_j\cos^j k\varphi+D_j\sin^j k\varphi) \tag{5}$$

With

$$d\varphi/d\xi=\sin k\varphi \tag{6}$$

Where A_0,A_l,B_l (l=1,2⋯m),C_0,C_j,D_j (j=1,2⋯n),k, u are undetermined constants. The parameter m n will be found by balancing the linear terms of highest order with the nonlinear terms.

Step 3: With the aid of symbolic computation in Matlab software , substituting (5) and (6) into(4) and collecting all terms with the same order of $\cos k\varphi,\sin k\varphi,\cos k\varphi\sin k\varphi$ will yield a set of algebra equations about trigonometric functions. The coefficients of each term in the trigonometric identity has to vanish.. This yields an overdetermined system of nonlinear algebraic equations with respect to A_0,A_l,B_l (l=1,2⋯m), C_0,C_j,D_j (j=1,2⋯n)

Step 4: Solving the above overdetermined system by applying Wu elimination method [12], we would yields the values of A_0,A_l,B_l (l=1,2⋯m), C_0,C_j,D_j (j=1,2⋯n) To equation (6), we can use separation of variables method to get its solutions

$$\cos k\varphi=-\tanh k(\xi+\xi_0),\sin k\varphi=\mathrm{sec}\,hk(\xi+\xi_0) \tag{7}$$

Where ξ_0 is integration constant. Finally, substituting(7) and values of m, n, $A_0,A_l,B_l\,(l=1,2,\cdots m),C_0,C_j,D_j\,(j=1,2,\cdots n),k,u$ into (5), we can obtain more general exact solutions of Eq. (4). In the next section, we study weak pinning charge density wave equation (1) of originated in the conductive polymer by using the trigonometric function transform method.

3. Exact solutions of charge density wave equations

Let us first consider Eq. (1) by using the above method. Substituting (3) into (1) and change Eq. (1) into the form

$$\left(u^2-c^2\right)\sigma_{\xi\xi}=a\sigma-b\sigma^3-b\sigma\rho^2;\left(u^2-c^2\right)\rho_{\xi\xi}=\left(a-4e\right)\rho-b\rho^3-b\rho\sigma^2 \tag{8}$$

Suppose the solutions of Eq. (8) have form (5) with (6). It is easy to show that m=n=1 by balancing $\sigma_{\xi\xi}$ with $\sigma^3,\sigma\rho^2$ and $\rho_{\xi\xi}$ with $\rho^3,\rho\sigma^2$ So we may choose

$$\sigma=A_0+A_1\cos k\varphi+B_1\sin k\varphi;\rho=C_0+C_1\cos k\varphi+D_1\sin k\varphi \tag{9}$$

Where $A_0,A_1,B_1,C_0,C_1,D_1,k,u$ are undetermined constants. From (6), (9) it is easy to deduce that

$$\sigma_{\xi\xi}=-2k^2A_1\cos k\varphi+k^2B_1\sin k\varphi+2k^2A_1\cos^3 k\varphi-2k^2B_1\sin^3 k\varphi \tag{10a}$$

$$\begin{aligned}\sigma^3&=A_0^3+(3A_0^2A_1+3A_1B_1^2)\cos k\varphi+(3A_0^2B_1+3A_1^2B_1)\sin k\varphi+3A_0A_1^2\cos^2 k\varphi\\&+3A_0B_1^2\sin^2 k\varphi+6A_0A_1B_1\cos k\varphi\sin k\varphi+(A_1^3-3A_1B_1^2)\cos^3 k\varphi+(B_1^3-3A_1^2B_1)\sin^3 k\varphi\end{aligned} \tag{10b}$$

$$\begin{aligned}\sigma\rho^2&=A_0C_0^2+(A_1C_0^2+2A_0C_0C_1+A_1D_1^2+2B_1C_1D_1)\cos k\varphi\\&+(B_1C_0^2+2A_0C_0D_1+B_1C_1^2+2A_1C_1D_1)\sin k\varphi+(A_0C_1^2+2A_1C_0C_1)\cos^2 k\varphi\\&+(A_0D_1^2+2B_1C_0D_1)\sin^2 k\varphi+(2A_0C_1D_1+2A_1C_0D_1+2B_1C_0C_1)\cos k\varphi\sin k\varphi\\&+(A_1C_1^2-A_1D_1^2-2B_1C_1D_1)\cos^3 k\varphi+(B_1D_1^2-B_1C_1^2-2A_1C_1D_1)\sin^3 k\varphi\end{aligned} \tag{10c}$$

$$\rho_{\xi\xi}=-2k^2C_1\cos k\varphi+k^2D_1\sin k\varphi+2k^2C_1\cos^3 k\varphi-2k^2D_1\sin^3 k\varphi \tag{10d}$$

$$\begin{aligned}\rho^3&=C_0^3+(3C_0^2C_1+3C_1D_1^2)\cos k\phi+(3C_0^2D_1+3C_1^2D_1)\sin k\varphi+3C_0C_1^2\cos^2 k\varphi+3C_0D_1^2\sin^2 k\varphi\\&+6C_0C_1D_1\cos k\varphi\sin k\varphi+(C_1^3-3C_1D_1^2)\cos^3 k\varphi+(D_1^3-3C_1^2D_1)\sin^3 k\varphi\end{aligned} \tag{10e}$$

$$\begin{aligned}\rho\sigma^2&=A_0^2C_0+(A_0^2C_1+2A_0A_1C_0+B_1^2C_1+2A_1B_1D_1)\cos k\varphi\\&+(A_0^2D_1+2A_0B_1C_0+A_1^2D_1+2A_1B_1C_1)\sin k\varphi+(A_1^2C_0+2A_0A_1C_1)\cos^2 k\varphi\\&+(B_1^2C_0+2A_0B_1D_1)\sin^2 k\varphi+(2A_1B_1C_0+2A_0B_1C_1+2A_0A_1D_1)\cos k\phi\sin k\varphi\\&+(A_1^2C_1-B_1^2C_1-2A_1B_1D_1)\cos^3 k\varphi+(B_1^2D_1-A_1^2D_1-2A_1B_1C_1)\sin^3 k\varphi\end{aligned} \tag{10f}$$

Substituting (9) and (10) into (8) and with the aid of symbolic computation in Matlab software, collecting all terms with the same order of $\cos k\varphi,\sin k\varphi,\cos k\varphi\sin k\varphi$, yields

$$\begin{aligned}&\left(u^2-c^2\right)\sigma_{\xi\xi}-a\sigma+b\sigma^3+b\sigma\rho^2=(-aA_0+bA_0^3+bA_0C_0^2)\\&+[-2k^2(u^2-c^2)A_1-aA_1+b(3A_0^2A_1+3A_1B_1^2)+b(A_1C_0^2+2A_0C_0C_1+A_1D_1^2+2B_1C_1D_1)]\cos k\varphi\\&+[k^2(u^2-c^2)B_1-aB_1+b(3A_0^2B_1+3A_1^2B_1)+b(B_1C_0^2+2A_0C_0D_1+B_1C_1^2+2A_1C_1D_1)]\sin k\varphi\\&+[3bA_0A_1^2+b(A_0C_1^2+2A_1C_0C_1)]\cos^2 k\varphi+[3bA_0B_1^2+b(A_0D_1^2+2B_1C_0D_1)]\sin^2 k\varphi\\&+[6bA_0A_1B_1+b(2A_0C_1D_1+2A_1C_0D_1+2B_1C_0C_1)]\cos k\varphi\sin k\varphi\\&+[2k^2(u^2-c^2)A_1+b(A_1^3-3A_1B_1^2)+b(A_1C_1^2-A_1D_1^2-2B_1C_1D_1)]\cos^3 k\varphi\\&+[-2k^2(u^2-c^2)B_1+b(B_1^3-3A_1^2B_1)+b(B_1D_1^2-B_1C_1^2-2A_1C_1D_1)]\cos^3 k\varphi=0\end{aligned} \tag{11a}$$

$$\begin{aligned}&\left(u^2-c^2\right)\rho_{\xi\xi}-\left(a-4e\right)\rho+b\rho^3+b\rho\sigma^2=[-(a-4e)C_0+bC_0^3+bA_0^2C_0]\\&+[-2k^2(u^2-c^2)C_1-(a-4e)C_1+b(3C_0^2C_1+3C_1D_1^2)+b(A_0^2C_1+2A_0A_1C_0+B_1^2C_1+2A_1B_1D_1)]\cos k\varphi\\&+[k^2(u^2-c^2)D_1-(a-4e)D_1+b(3C_0^2D_1+3C_1^2D_1)+b(A_0^2D_1+2A_0B_1C_0+A_1^2D_1+2A_1B_1C_1)]\sin k\varphi\\&+[3bC_0C_1^2+b(A_1^2C_0+2A_0A_1C_1)]\cos^2 k\varphi+[3bC_0D_1^2+b(B_1^2C_0+2A_0B_1D_1)]\sin^2 k\varphi\\&+[6bC_0C_1D_1+b(2A_1B_1C_0+2A_0B_1C_1+2A_0A_1D_1)]\cos k\varphi\sin k\varphi\\&+[2k^2(u^2-c^2)C_1+b(C_1^3-3C_1D_1^2)+b(A_1^2C_1-B_1^2C_1-2A_1B_1D_1)]\cos^3 k\varphi\\&+[-2k^2(u^2-c^2)D_1+b(D_1^3-3C_1^2D_1)+b(B_1^2D_1-A_1^2D_1-2A_1B_1C_1)]\sin^3 k\varphi=0\end{aligned} \tag{11b}$$

Setting the coefficients of each order of $\cos k\varphi,\sin k\varphi,\cos k\varphi\sin k\varphi$ in (11) to zero, yields a set of algebraic equations with respect to the unknowns $A_0,A_1,B_1,C_0,C_1,D_1,k$

$$-aA_0+bA_0^3+bA_0C_0^2=0$$

$$-2k^2(u^2-c^2)A_1 - aA_1 + b(3A_0^2A_1 + 3A_1B_1^2) + b(A_1C_0^2 + 2A_0C_0C_1 + A_1D_1^2 + 2B_1C_1D_1) = 0$$
$$k^2(u^2-c^2)B_1 - aB_1 + b(3A_0^2B_1 + 3A_1^2B_1) + b(B_1C_0^2 + 2A_0C_0D_1 + B_1C_1^2 + 2A_1C_1D_1) = 0$$
$$3bA_0A_1^2 + b(A_0C_1^2 + 2A_1C_0C_1) = 0; 3bA_0B_1^2 + b(A_0D_1^2 + 2B_1C_0D_1) = 0$$
$$6bA_0A_1B_1 + b(2A_0C_1D_1 + 2A_1C_0D_1 + 2B_1C_0C_1) = 0$$
$$2k^2(u^2-c^2)A_1 + b(A_1^3 - 3A_1B_1^2) + b(A_1C_1^2 - A_1D_1^2 - 2B_1C_1D_1) = 0$$
$$-2k^2(u^2-c^2)B_1 + b(B_1^3 - 3A_1^2B_1) + b(B_1D_1^2 - B_1C_1^2 - 2A_1C_1D_1) = 0$$
$$-(a-4e)C_0 + bC_0^3 + bA_0^2C_0 = 0$$
$$-2k^2(u^2-c^2)C_1 - (a-4e)C_1 + b(3C_0^2C_1 + 3C_1D_1^2) + b(A_0^2C_1 + 2A_0A_1C_0 + B_1^2C_1 + 2A_1B_1D_1) = 0$$
$$k^2(u^2-c^2)D_1 - (a-4e)D_1 + b(3C_0^2D_1 + 3C_1^2D_1) + b(A_0^2D_1 + 2A_0B_1C_0 + A_1^2D_1 + 2A_1B_1C_1) = 0$$
$$3bC_0C_1^2 + b(A_1^2C_0 + 2A_0A_1C_1) = 0; 3bC_0D_1^2 + b(B_1^2C_0 + 2A_0B_1D_1) = 0$$
$$6bC_0C_1D_1 + b(2A_1B_1C_0 + 2A_0B_1C_1 + 2A_0A_1D_1) = 0$$
$$2k^2(u^2-c^2)C_1 + b(C_1^3 - 3C_1D_1^2) + b(A_1^2C_1 - B_1^2C_1 - 2A_1B_1D_1) = 0$$
$$-2k^2(u^2-c^2)D_1 + b(D_1^3 - 3C_1^2D_1) + b(B_1^2D_1 - A_1^2D_1 - 2A_1B_1C_1) = 0 \quad (12)$$

With the aid of symbolic computation in Matlab software, solving the above over-determined algebraic equations (12) by applying Wu elimination method [12], and the following general solutions were obtained:

$$\text{(i)}\ A_0 = C_0 = B_1 = C_1 = 0, A_1 = \pm\sqrt{a/b}, D_1 = \pm\sqrt{(a-8e)/b}, k = \pm\sqrt{4e/(c^2-u^2)} \quad (13a)$$
$$\text{(ii)}\ A_0 = C_0 = A_1 = D_1 = 0, B_1 = \pm\sqrt{(a+4e)/b}, C_1 = \pm\sqrt{(a-4e)/b}, k = \pm\sqrt{4e/(u^2-c^2)} \quad (13b)$$
$$\text{(iii)}\ A_0 = C_0 = B_1 = D_1 = 0, A_1^2 + C_1^2 = a/b, A_1 = \pm\sqrt{a/b}\cos\alpha, C_1 = \pm\sqrt{a/b}\sin\alpha, k = \pm\sqrt{a/2(c^2-u^2)} \quad (13c)$$
$$\text{(iv)}\ A_0 = C_0 = A_1 = C_1 = 0, B_1^2 + D_1^2 = a/b, B_1 = \pm\sqrt{2a/b}\cos\beta, D_1 = \pm\sqrt{2a/b}\sin\beta, k = \pm\sqrt{a/(u^2-c^2)} \quad (13d)$$

In (13c) and (13d), e need to satisfy the restriction condition to Eqs.(1): e=0. Combining (3) and (7), substituting (13) into (9), we find the following a series of exact solutions for charge density wave equations (1) in conductive polymer

(a) $$\sigma_1(x,t) = \pm\sqrt{a/b}\tanh\left[\sqrt{4e/(c^2-u^2)}\left(x-ut+x_0\right)\right]; \rho_1(x,t) = \pm\sqrt{(a-8e)/b}\,\mathrm{sec}h\left[\sqrt{4e/(c^2-u^2)}\left(x-ut+x_0\right)\right] \quad (14)$$

When $ab > 0, (a-8e)b > 0, e(c^2-u^2) > 0$ (14) are exact traveling wave soliton solutions of Eqs.(1), and when $ab < 0, (a-8e)b > 0, e(c^2-u^2) < 0$ (14) become exact traveling wave trigonometric function solutions of Eqs.(1), namely

$$\sigma_2(x,t) = \pm\sqrt{-a/b}\tan\sqrt{4e/(u^2-c^2)}\left[\left(x-ut+x_0\right)\right]; \rho_2(x,t) = \pm\sqrt{(a-8e)/b}\sec\left[\sqrt{4e/(u^2-c^2)}\left(x-ut+x_0\right)\right] \quad (15)$$

(b) $$\sigma_3(x,t) = \pm\sqrt{(a+4e)/b}\,\mathrm{sec}h\left[\sqrt{4e/(u^2-c^2)}\left(x-ut+x_0\right)\right]; \rho_3(x,t) = \pm\sqrt{(a-4e)/b}\tanh\left[\sqrt{4e/(u^2-c^2)}\left(x-ut+x_0\right)\right] \quad (16)$$

When $(a+4e)b > 0, (a-4e)b > 0, e(u^2-c^2) > 0$ (16) are exact traveling wave soliton solutions of Eqs.(1), and when $(a+4e)b > 0, (a-4e)b < 0, e(u^2-c^2) < 0$ (16) become exact traveling wave trigonometric function solutions of Eqs.(1), namely

$$\sigma_4(x,t) = \pm\sqrt{(a+4e)/b}\sec\left[\sqrt{4e/(c^2-u^2)}\left(x-ut+x_0\right)\right]; \rho_4(x,t) = \pm\sqrt{(4e-a)/b}\tan\left[\sqrt{4e/(c^2-u^2)}\left(x-ut+x_0\right)\right] \quad (17)$$

(c) $$\sigma_5(x,t) = \pm(\sqrt{a/b}\cos\alpha)\tanh\left[\sqrt{a/2(c^2-u^2)}\left(x-ut+x_0\right)\right]; \rho_5(x,t) = \pm(\sqrt{a/b}\sin\alpha)\tanh\left[\sqrt{a/2(c^2-u^2)}\left(x-ut+x_0\right)\right] \quad (18)$$

When $ab > 0, a(c^2-u^2) > 0, e = 0$ (18) are exact traveling wave soliton solutions of Eqs.(1), and when $ab < 0, a(c^2-u^2) < 0, e = 0$ (18) become exact traveling wave trigonometric function solutions of Eqs.(1), namely

$$\sigma_6(x,t) = \pm(\sqrt{-a/b}\cos\alpha)\tan\left[\sqrt{a/2(u^2-c^2)}\left(x-ut+x_0\right)\right]; \rho_6(x,t) = \pm(\sqrt{-a/b}\sin\alpha)\tan\left[\sqrt{a/2(u^2-c^2)}\left(x-ut+x_0\right)\right] \quad (19)$$

(d) $$\sigma_7(x,t) = \pm(\sqrt{2a/b}\cos\beta)\,\mathrm{sec}h\left[\sqrt{a/(u^2-c^2)}\left(x-ut+x_0\right)\right]; \rho_7(x,t) = \pm(\sqrt{2a/b}\sin\beta)\,\mathrm{sec}h\left[\sqrt{a/(u^2-c^2)}\left(x-ut+x_0\right)\right] \quad (20)$$

When $ab > 0, a(u^2 - c^2) > 0, e = 0$ (20) are exact traveling wave soliton solutions of Eqs.(1), and when $ab > 0, a(u^2 - c^2) < 0, e = 0$ (20) become exact traveling wave trigonometric function solutions of Eqs.(1), namely

$$\sigma_8(x,t) = \pm(\sqrt{2a/b}\cos\beta)\sec\left[\sqrt{a/(c^2-u^2)}\left(x-ut+x_0\right)\right]; \rho_8(x,t) = \pm(\sqrt{2a/b}\sin\beta)\sec\left[\sqrt{a/(c^2-u^2)}\left(x-ut+x_0\right)\right] \quad (21)$$

In addition, in the above four cases, according to different values of a,b,c,e,u we easy obtain plural form exact solutions for equations (1), namely

$$\sigma_9(x,t) = \pm i\sqrt{-a/b}\tanh\left[\sqrt{4e/(c^2-u^2)}\left(x-ut+x_0\right)\right]; \rho_9(x,t) = \pm i\sqrt{(8e-a)/b}\,\mathrm{sec}h\left[\sqrt{4e/(c^2-u^2)}\left(x-ut+x_0\right)\right] \quad (22)$$

$$\sigma_{10}(x,t) = \pm i\sqrt{-(a+4e)/b}\,\mathrm{sec}h\left[\sqrt{4e/(u^2-c^2)}\left(x-ut+x_0\right)\right]; \rho_{10}(x,t) = \pm i\sqrt{-(a-4e)/b}\tanh\left[\sqrt{4e/(u^2-c^2)}\left(x-ut+x_0\right)\right] \quad (23)$$

$$\sigma_{11}(x,t) = \pm(i\sqrt{-a/b}\cos\alpha)\tanh\left[\sqrt{a/2(c^2-u^2)}\left(x-ut+x_0\right)\right]; \rho_{11}(x,t) = \pm(i\sqrt{-a/b}\sin\alpha)\tanh\left[\sqrt{a/2(c^2-u^2)}\left(x-ut+x_0\right)\right] \quad (24)$$

$$\sigma_{12}(x,t) = \pm(i\sqrt{-2a/b}\cos\beta)\mathrm{sec}h\left[\sqrt{a/(u^2-c^2)}\left(x-ut+x_0\right)\right]; \rho_{12}(x,t) = \pm(i\sqrt{-2a/b}\sin\beta)\mathrm{sec}h\left[\sqrt{a/(u^2-c^2)}\left(x-ut+x_0\right)\right] \quad (25)$$

4. Conclusions

In this paper, we have obtained a series of exact solutions (including four kinds of soliton solutions and four plural form exact solutions) for weak pinning charge density wave equations (1) by using the trigonometric function transform method. At the same time, some new trigonometric function solutions are found. The main idea of our method is that assumed the unknown solutions are the trigonometric function polynomial form. The method is a direct and systematic method with general rules. Thus it can widely also be applied to solving more nonlinear evolution differential equation or nonlinear coupled evolution equations originated in nonlinear materials sciences and engineering.

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